On the Failure of the Linkage Principle with Colluding Bidders*

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Abstract

One of the fundamental results of auction theory is the Linkage Principle, which states that the seller's expected revenue is enhanced by *ex ante* full public disclosure of information about the value of the good. Previous literature has established that the Principle fails in dynamic settings. We argue that information provision may also sustain collusion in single-unit auctions, thus harming the auctioneer. Using a model with private and common value components, we point out that disclosure curtails common value uncertainty, making communication between cartel members incentive compatible. We also show that collusion is feasible if and only if information provision reaches an interior threshold value.

JEL Codes: C72, D44, D82, L41
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1 Introduction

In auctions with non-cooperative bidders, transparency decreases information asymmetry between market actors, and thereby enhances competition. This result, attributed to Milgrom and Weber (1982), is often cited as the Linkage Principle. Milgrom and Weber (1982), Goeree and Offerman (2003) and Fatima et al. (2005) point out, that lower common value (CV) uncertainty increases expected revenue. They argue that the reduction of uncertainty by public disclosure results in lower information rents and reduces the extent of the winner's curse. Therefore, the optimal choice of a risk-neutral seller is full disclosure. We show that this fundamental result does not extend to auctions exposed to cartel agreements.

Transparency is not necessarily the best policy and it is discouraged in procurement auctions in order to combat collusion (OECD, 2014). While international procurement guidelines recognize the negative effect of public disclosure of information, there is no formal theory regarding this matter. Collusion is a major concern in auctions (Marshall et al., 2014). Kawai and Nakabayashi (2015) estimate that, in Japan, nearly 20 percent of the total contract value was awarded in non-competitive public tenders.

We focus on disclosure in the context of sealed-bid auctions for a single indivisible item. Public disclosure of information by the seller helps cartel formation by enabling an incentive compatible collusive mechanism. Collusion reduces revenue by bid suppression, so that the effect of public disclosure decreases the seller's profit.

In an auction context, public disclosure reduces the uncertainty about the valuation of all bidders. Procurement auctions involve cost factors, about which the auctioneer has private information. A number of government agencies routinely prepare independent cost estimates (FTA, 2013). The European Union imposes a mandatory contract value estimate before the start of the procurement process following Directive 2014/24/EU.[1]

Bidders also face idiosyncratic costs and preferences that are unaffected by disclosure policy. We model this dichotomy with CV and private value (PV) uncertainty.[2] A model including PV and CV information asymmetries allows us to consider a seller optimizing over disclosure. The pure independent

[1] Council of the European Union (2014).

[2] While the vast majority of research employs only pure models, the assumption that values are exclusively private or common rarely holds in actual auctions (Laffont, 1997; Goeree and Offerman, 2003).

PV model assumes that valuations are conditionally independent. Thus, a seller possesses no private information about buyers' valuations. In a CV framework, collusion is only possible under fairly strong conditions. There exists a collusive mechanism if the cartel is able to communicate, members weakly prefer collusion and the cartel has full control over members' bids (McAfee and McMillan, 1992).

Our model considers a one-shot sealed-bid auction setting for a single, indivisible good.[3] The valuation of bidders is modeled by additively separable, binary private and common value elements. Disclosure is modeled as changing the distribution of CV while keeping *ex ante* expected valuation of bidders constant.

Related literature emphasizes the negative effects of disclosure in auctions by focusing exclusively on its dynamic effects. Marshall and Marx (2009) analyze a one-shot independent private values (IPV) auction with a registration process. They point out that a seller is able to reduce the cartel's revenue by choosing a less transparent regime of participant registration. Ascending-bid auctions are susceptible to collusion if participants are identifiable. Samkharadze (2012) addresses the problem in a two-stage procurement setting in which the buyer is able to reveal private information to the sellers between stages. The policy of public information revelation decreases expected payoff if bidders form a ring. [4]

We borrow the concept of bid coordination mechanism (BCM) from Marshall and Marx (2007). That is, ring members are able to communicate and send side-payments to each other. We show that, in an IPV model, the cartel is able to form and bidders truthfully reveal their types. Such mechanisms are available in the form of a pre-auction knockout, in which members bid for the right to bid for the good and determine side-payments. In a knock-out auction, the member with the highest valuation will be the designated bidder. Incentive compatibility is ensured by the availability of side-payments and the lower price resulting from the other bidders suppressing their bids. For CV, no incentive compatible mechanism is available. In this setting, collud-

[3]One-shot auctions are often subjected to collusive schemes, as evidenced in US v. WF Brinkley & Son Const. Co. (1986), US v. AAA Elec. Co. Inc. (1986), US v. Metropolitan Enterprises Inc. (1984), US v. Reicher (1992), US v. MMR Corp. (1992), US v. Rose (2006), US v. Green (2010) and US, EX REL. McGEE v. IBM Corporation (2015).

[4]While it is not the focus of our study, the Linkage Principle can be violated in multi-unit auctions (Perry and Reny, 1999) and if bidders are financially constrained (Fang and Parreiras, 2003).

ing players share the same information set and the same expected valuation. Therefore, there are strong incentives to misreport one's type. Information pooling does not help in choosing the efficient buyer. Consequently, incentive compatibility of any mechanism is problematic.

If an incentive compatible mechanism is available, the ring is able to maximize its surplus by choosing the member with the highest PV to bid. The other advantage of a collusive agreement is information pooling. Accordingly, the possibility of sharing CV signals is increasing the incentives to form a bidding ring. Although we do not dispute this notion, we point out that higher CV variance is able to destroy a collusive agreement. With higher variance of the CV term, a bidder is able to alter its report to the cartel to a larger extent. Hence, it is able to manipulate the designated bid. The effect of higher CV uncertainty destroys the incentive compatibility of the mechanism, since participants can anticipate this behavior.

In the framework of Milgrom and Weber (1982), disclosure not only allows the seller to reduce common uncertainty, but also brings the auction market closer to the IPV model. Our model illustrates that it also facilitates collusion, and claims that the Linkage Principle does not hold if bidders can engage in a conspiracy. In the spirit of Laffont and Martimort (1997) we emphasize that group incentives are crucial in understanding the effect of information disclosure. Section 2 builds up the framework of a hybrid auction model and derives the necessary and sufficient conditions for the existence of incentive compatible collusive mechanisms. The results apply to a class of sealed-bid auction mechanisms. We focus on the ability of the bidding ring to suppress internal competition. In Section 3, we apply these results to second-price auctions, and we provide an example of the disclosure effect on collusion.

In our model with collusive bidders, revenue is not an increasing function of the information available, as opposed to the non-cooperative model analyzed by Milgrom and Weber (1982) and Goeree and Offerman (2003). In Section 4 we prove that the pure private and common value models are robust to perturbation by the other source of information asymmetry. Thus, if we introduce small common value perturbations in a pure private value model, collusion remains feasible. Similarly, there is no incentive compatible bid coordination mechanism in the neighborhood of a pure common value model. Section 5 shows that the well-established result of increasing revenue with respect to common value uncertainty does not hold if bidders can form a bidding ring. Finally, Section 6 concludes.

2 Collusion in a hybrid model

This section constructs an auction model with additively separable values applying the model of McLean and Postlewaite (2004).[5] In the present context, we refer to this setup as the hybrid model, following Milgrom and Weber (1982). Common uncertainty experienced by all players can be modeled by the distribution of the common value, which is taken by bidders as given. A number of bidders may form a bidding ring before participating in a sealed-bid auction. Subsection 2.1 constructs the model. The concept of incentive compatible collusive mechanisms is introduced in Subsection 2.2 where we characterize its existence. In all cases, we consider perfect Bayesian Nash equilibria (PBNE).

2.1 Valuation of bidders

Risk-neutral and symmetric players bid for a single commodity. Bidder i receives a two-dimensional signal (x_i, y_i), where x_i is the independent private value (PV) component of a bidder's valuation, and it is a random variable with discrete probability distribution $x_i \in \{x_L, x_H\}$ with equal probabilities, $x_L \leq x_H$.

The common value (CV) y is drawn from $\{y_L, y_H\}$ with equal probabilities, where $y_L < y_H$. Signal y_i is observed by bidder i, which can take $y_i \in \{y_L, y_H\}$ where $y_i = y$ with probability $\delta \in \left(\frac{1}{2}, 1\right)$. For any y and for $i \neq j$, y_i and y_j are conditionally independent. Valuation v_i of bidder i is equal to the sum of her PV signal and the CV, $v_i = x_i + y$.[6] Hence, bidders face common uncertainty about their valuations and individual valuations can be different.[7]

[5] This is a standard assumption for private and common values. Pesendorfer and Swinkels (2000), Goeree and Offerman (2003) and Fatima et al. (2005) also analyze models with additively separable values.

[6] Note, that our model does not include pure CV auctions. The reason is that collusion is supported in that case by a random allocation of the good between ring members, as pointed out by McLean and Postlewaite (2004).

[7] This paper models CV uncertainty as the spread of CV types. We can note that δ also captures uncertainty and it is able to model disclosure. Changing δ has two effects. Extreme values $\delta = 0.5$ and $\delta = 1$ both represent a signal carrying no CV asymmetry between bidders and expected payoffs are identical to risk-neutral bidders. So, there is collusion in the neighborhood of extreme values, as illustrated in Figure 1. Most importantly, it also has a non-monotonic effect on expected revenue. Too high δ induces collusion and results

This model can be linked to the hybrid model by Milgrom and Weber (1982). They assume that bidders' private information can be expressed by single real-valued informational variables which have affiliated densities.[8] The construction of parameter δ ensures positive affiliation. If someone receives a high signal, the conditional expected value of another player's signal is also higher. We can define an informational variable simply as the sum of signals, which generally identifies both components.[9] Discrete distribution of both components allows for a solution for the single-valued representation.

We consider two standard auction mechanisms. Both are sealed-bid formats, bids are submitted simultaneously. A bid is a non-negative value b_i by which the player submitting the highest bid wins a non-divisible commodity. The price is paid only by the winner. In the first-price auction, this equals the bid of the winner. In the second-price auction, this is equal to the second highest bid.[10]

2.2 Equilibrium with bid coordination mechanism

A collusive mechanism is a function determining a bidding strategy and side-payments among ring members, conditional on signals they send to each other prior to the auction. Models of collusive mechanisms in auctions distinguish cartel types according to their ability to communicate, to verify information, to make transfer payments and to control bids (McAfee and McMillan, 1992). We apply the concept of bid coordination mechanism (BCM) (Marshall and Marx, 2007; Lopomo et al., 2011). A BCM allows for pre-auction

in a negative drop at this point of discontinuity.

[8]We define affiliation following the simple definition of Castro (2010). Although this defines affiliation for density functions, we can generalize it for any probability distribution. We say that the density function $f : [\underline{t}, \bar{t}] \rightarrow \mathbb{R}_+$ is affiliated, if for any t, t', we have $f(t) f'(t) \leq f(t \wedge t') f(t \vee t')$, at which $t \wedge t' = (min\{t_1, t_1'\}, \ldots, min\{t_n, t_n'\})$ and $t \vee t' = (max\{t_1, t_1'\}, \ldots, max\{t_n, t_n'\})$. The concept is called Multivariate Total Positivity of Order 2 (MTP2) for the multivariate case by Karlin (1968).

[9]Milgrom and Weber (1982) assume that individual valuations are determined by informational variables and a number of non-observed variables. We can construct these non-observables as the difference between informational variables and real individual valuations resulting in: $w_i = y - y_i$.

[10]We do not directly address the problem of setting a reserve price. While it is relevant in a pure independent PV model, Levin and Smith (1996) show that the revenue-maximizing reserve price monotonically and often rapidly converges to the seller's valuation as the number of bidders grows.

side-payments[11] and a set of recommended bids as a function of signals about ring member types. Side-payments serve as an incentive device in setting lacking repeated interaction.[12]

Surplus of bid rigging comes from suppressing competition and pooling available information. Pre-auction transfers are necessary in order to incentivize ring members.[13] On the other hand, punishment for deviating from the cartel agreement can be costly.[14] We also assume recommended bids cannot be enforced by the ring.[15]

The majority of theoretical models consider incentive compatible collusive mechanisms with symmetric bidders (Graham and Marshall, 1987; McAfee and McMillan, 1992; Marshall and Marx, 2007). We also also assume players are *ex ante* symmetric with respect to information variables (x_i, y_i). Following Marshall and Marx (2007), we consider an exogenously determined ring of $n \geq 2$ members, where the set of cartel members is denoted by N. There are $k \geq 0$ outsiders. If a ring faces at least one outsider, it is called a non-inclusive ring. All-inclusive rings encompass all players. We assume the set of players and ring membership are exogenously given.

An outsider bidder j, if any, plays according to a given pure strategy $\alpha_j(x_j, y_j)$. We assume the only arguments of this function are the informational variables observed by the player. Also, the outsider is not a strategic player in the sense that her strategy is independent of the existence of the ring and it is not necessarily a best response to the ring members' strategies.[16] Nevertheless, cartel members' strategy maximize their expected payoff considering the outsider strategy. In what follows we apply the simplified

[11]McAfee and McMillan (1992) model *ex post* knockout auctions, which are common in practice. However, they are subjected to *ex post* inefficiency, that is, the designated bidder might post a bid higher than any bid in the knockout.

[12]For a list of cartel cases involving side-payments see Marshall and Marx (2009).

[13]In practice, transfers typically come indirectly. Kovacic et al. (2006) emphasizes they often come in the form of subcontracts.

[14]For a study on the applied model with possibility of *ex post* actions, see Marshall and Marx (2009).

[15]Enforcement can come from punishment mechanism either externally (organized crime) or in the form of a grim trigger strategy (McAfee and McMillan, 1992; Mailath and Zemsky, 1991). Since all these examples stem from a repeated game, it is arguable that such tools are not available for the ring in a one-shot setup. Another standard way is to employ an agent submitting all bids. This is difficult to organize, since it assumes anonymity. Asker (2010) demonstrates this on a stamp-dealer cartel which participated in open auctions with no legal entry barriers.

[16]An outsider with hard evidence can turn to the authorities.

notation of $\alpha(\cdot)$.

Formally, a BCM is a function

$$\mu(x^*, y^*) = (\beta(x^*, y^*), p(x^*, y^*))$$

chosen by the "center", a standard incentiveless mechanism agent, where (x^*, y^*) denotes the vector of signals simultaneously shared within the ring, indicating PV and CV signals. Vector $\beta(\cdot)$ represents recommended bids and $p(\cdot)$ is the normalized side-payment vector. That is, $p_i(\cdot)$ is the amount ring member i receives from other members, and the sum of components is $\sum p_i(\cdot) = 0$, satisfying *ex post* budget balance. The timing of the game is as follows.

1. Ring members learn mechanism $\mu(\cdot)$ chosen by the center.
2. They make a decision about participation.
3. They learn their types (x_i, y_i).
4. If a ring is formed, members share signals (x^*, y^*) simultaneously. Following the mechanism, members learn the recommended bids $\beta(x^*, y^*)$, side-payments $p(x^*, y^*)$ are enforced and implemented.[17]
5. Players submit their bids in the auction.

In what follows we define BCM. Function Π_i denotes the expected payoff (with side-payments) of ring member i. We say that $\mu(\cdot)$ is a BCM against outside bid function $\alpha(\cdot)$, if conditions (1), (2) and (3) hold. We denote expected values over all bidder types with $E(\cdot)$, all payoffs and subscripts refer to ring members, subscript $-i$ refers to members of the ring other than member i.

$$(x_i, y_i) \in arg\,max_{x_i^*, y_i^*}\, \mathbb{E}\left(\Pi_i(\cdot)\,|\mu(\cdot), x_{-i}^* = x_{-i}, y_{-i}^* = y_{-i}, \alpha(\cdot)\right), \forall i \qquad (1)$$

$$\beta_i(x^*, y^*) \in arg\,max\, \mathbb{E}\left(\Pi_i(\cdot)\,|x^*, y^*, \beta_{-i}, \alpha(\cdot)\right) \qquad (2)$$

[17] *Ex ante* implementation of side-payments is important to avoid costly re-negotiations and rent-seeking Marshall and Marx (2012).

$$\beta(x^*, y^*) \in \arg\max \mathbb{E}\left(\sum_i \Pi_i(\cdot) \,|x^*, y^*, \alpha(\cdot)\right) \tag{3}$$

Thus, incentive compatibility has the following requirements. Condition (1) requires that members find it optimal to truthfully reveal their types. Condition (2) captures the idea that recommended bids are not enforced, following them must be optimal for ring members. Finally, condition (3) concerns the optimal collusive strategy, which is achieved if the sum of their payoffs is maximal. We say that the ring is able to suppress all ring competition in that case. Our definition of BCM differs from Marshall and Marx (2007), in that we also consider all-inclusive rings.[18] Additionally, note that our definition requires that the mechanism is optimal for the ring. The reason behind this consideration is that allocative efficiency in a setting involving both CV and PV is not guaranteed by the other conditions. A ring member with the highest PV must have the highest recommended bid to ensure efficiency.

In the spirit of Marshall and Marx (2007), the mechanism does not involve randomization, except for a tie-breaking rule. If a number of mechanisms $\mu(\cdot)$ are permutations of side-payments and recommended bids, and they provide the same expected payoff for the ring, a mixed mechanism is applied, in which one of them are chosen randomly with the same probability. It is easy to see there is a finite number of permutations and this happens if and only if the permutation is between members who report the same PV, following Condition (3).

Additionally, we only consider individually rational mechanisms.[19] That is, in which participation provides higher *ex ante* expected payoff than the competitive game.[20] Ring members coordinate their bids. This includes the possibility of the competitive equilibrium strategy. Therefore, the sum of the ring members' payoffs is at least as high as without the collusive mechanism.[21]

[18]In case there are no outsiders, we shall assume an outside bid function $\alpha(\cdot) = 0$.

[19]In other words, it needs to satisfy the weak participation constraint, as defined by Borgers et al. (2015).

[20]For the latter one we consider a Bayesian Nash equilibrium (BNE) in which outsiders are non-strategic. As define before, they play according to $\alpha(x_j, y_j)$. In Section 3, we show an example in which one outsider plays according to the symmetric BNE of the competitive game, but in what follows we do not make this assumption.

[21]The participation constraint holds, if outsiders are strategic and they bid their best

Consequently, any BCM satisfies the *ex ante* participation constraint.

BCM is a direct mechanism. This consideration is followed by the Revelation Principle for BNE (Fudenberg and Tirole, 1991). That is, if a BNE implements a certain choice, it is also truthfully implementable. At this point we assume distribution of CV signals is given. The problem of the seller is addressed in Section 5.

Due to *ex ante* symmetry, they make a unanimous decision, so we only address mechanisms in which all n members participate. It follows from Condition (3) that, if there is a BCM, there exists a mechanism in which all members submit 0, except for one. We can further restrict our attention to a subset of incentive compatible mechanisms, as pointed out in Lemma 2.1.

Lemma 2.1. *If the set of incentive compatible BCMs is non-empty, at least one of them satisfies that the designated bidder is the member with the highest PV.*

Proof. A BCM $\mu(\cdot)$ allots recommended bids to the ring. We refer to the ring member with a positive bid as designated bidder. This is the member with the highest PV (without loss of generality, member 1). Consider $\mu(\cdot)$, incentive compatible, in which there is at least one pair $(x^*, y^*), \left(x^*, y^{*\prime}\right)$, such that the designated bidders are different. That is, there is at least one of these pairs in which the designated bidder is not the one with the highest PV (member 2). Payoffs can be weakly improved by switching the designated bidder's role to bidder 1 in mechanism $\mu'(\cdot)$. The side-payment of player 2 shall be equal to the expected profit of being a designated player according to mechanism $\mu(\cdot)$. That is, all surplus from choosing the efficient buyer goes to player 1 in mechanism $\mu'(\cdot)$. Thus, $\mu'(\cdot)$ also results in an incentive compatible solution, since all constraints remain identical. □

This consideration of restricting attention to the PV here stems from the information pooling function of the ring. If someone with a strictly lower PV becomes the designated bidder, her partner with higher value would have higher payoff, consequently, she would have incentives to bid higher than her partner in the auction. We point out that there is usually a continuum of BCMs, if any. The non-designated player can submit a sufficiently low bid,

response with respect to the ring. In that case, outsiders bid less aggressively, which increases the ring's payoff.

which does not increase the expected payment, conditional on winning.[22]

A BCM defines a side-payment vector p of dimension $n \cdot 2^{2n}$. The n ring members send $2n$ signals, and all of them can attain two possible values. This defines 2^{2n} profiles, which are applied to all ring members. Similarly, the recommended bid vector can be expressed as a vector of $n \cdot 2^{2n}$ dimensions. There is a designated bidder who is randomly chosen among the bidders with the highest PV.

Lemma 2.2. *Suppose that the set of BCMs M_D is non-empty. Then, if we take $\mu = \left(\hat{p}, \hat{\beta}\right) \in M_D$, the set of p for which $\mu = \left(p, \hat{\beta}\right) \in M_D$ is convex.*

Proof. Given that there is a designated bidder receiving an optimal recommended strategy, all types shall be truthfully revealed according to Condition (1). That is, no type finds it better to misreport. There are 4 possible types, each defining 3 incentive compatibility constraints, together 12. On both sides of this equation, the payoffs are a linear function of side-payment components in p. Since $\mathbb{R}^{n \cdot 2^{2n}}$ is convex, the resulting set is also convex.[23] □

Lemma 2.2 characterizes the set of incentive compatible mechanisms. Convexity implies that if two BCMs with the same recommended bid function are incentive compatible, so are their convex combinations. Ring members are *ex ante* symmetric, so incentive compatibility is maintained if we permute them. The linear combination of such mechanisms results in a symmetric mechanism with respect to the ring members. That is, if M_D is non-empty, there exists a mechanism so that the side-payment only depends on the number of high private and common value reports within the ring and the own type.

Accordingly, we can apply the notation $p\left(|x_H|, |y_H|\right)$, which is the sum of the amounts that bidders with low reported PV receive. Similarly, an equal aggregate amount is subtracted from those who have high PV. Since there are $n - |x_H|$ members with low and $|x_H|$ members with high PV, the side-payment of each ring member with high PV equals $-\frac{1}{|x_H|} p\left(|x_H|, |y_H|\right)$. Similarly, the same amount for members with low PV report equals $\frac{1}{n-|x_H|} p\left(|x_H|, |y_H|\right)$. The sum of these values equals zero, so *ex post* budget balance is satisfied.

[22] Any value between 0 and the lowest equilibrium outside bid can be a complementary or cover bid if it does not affect the price.

[23] Intersection of convex sets is always convex (Simon et al., 1994).

To sum up, if there is an incentive compatible mechanism, there is one in which the side-payment and the recommended bid of a member only depends on the number of certain signals within the group and the own PV. In addition to the PV of the designated bidder, distribution of CV signals determine the maximal expected gain from participating in the auction.

The existence of a BCM depends on the auction format and the extent of CV uncertainty. Our points are formally stated in Proposition 2.3 and 2.4, which serve as the main results of our paper for second- and first-price auctions, respectively. Our propositions serve as an extension of the results of Marshall and Marx (2007). First, they point out that CV variance affects the existence of BCM. Second, conditionally on the existence of incentive compatible BCM, they confirm the results hold for positive CV variance.

Without loss of generality, the designated bidder is denoted by 1, whereas index -1 refers to non-designated ring members. The expected payoffs always use the following notations. Function $\pi\left(x_1, |y_H|, \alpha\left(\cdot\right)\right)$ represents the expected payoff of a bidder as a function of her own PV and the number of high CV signals among $n-1$ other bidders, who submit zero bid. The remaining bidders are assumed to follow strategy $\alpha\left(\cdot\right)$, as defined earlier. Function $\pi\left(x_1, |y_H|, \alpha\left(\cdot\right)\right)$ only takes the outcome of the auction into account. That is, the designated bidder's total expected payoff is $\pi\left(x_1, |y_H|, \alpha\left(\cdot\right)\right) - p\left(|x_H|, |y_H|\right)$, conditional on truthfully reported signals.

Proposition 2.3. *Suppose there is a given bidding ring with n members and $k \geq 0$ outsiders in a sealed-bid second-price auction. There exists a BCM if and only if* (4) *is satisfied.*[24]

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_L\right)\frac{1}{i}\pi\left(x_1 = x_H, |y_H| = j, \alpha\left(\cdot\right)\right) \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_H\right)\frac{1}{i+1}\pi\left(x_1 = x_L, |y_H| = j, \alpha\left(\cdot\right)\right) \quad (4)$$

Proof. See Appendix A.[25] □

[24] On the RHS we can see that the expected revenue function captures a case when the designated bidder has low PV while the number of ring members with high PV's is positive. This side of the inequality comes from an incentive compatibility constraint capturing misreported PV type. See Appendix A.

[25] Appendix A also provides an equivalent form of the inequality (4), with probability values written explicitly.

Proposition 2.3 provides a necessary and sufficient condition for the existence of an incentive compatible BCM in second-price auctions. This result can be interpreted as follows. There exists a BCM if and only if the relative CV uncertainty is greater than the PV uncertainty. This is highlighted by the two sides of (4). The left-hand side (LHS) of the inequality employs high own PV and lower CV signals, while the right-hand side (RHS) has low PV and higher CVs. If the relative CV uncertainty becomes greater, collusion breaks down.

On the contrary, if (4) is not satisfied, there is no incentive compatible side-payment vector. Here side-payments must satisfy a two-fold role: providing incentives not to overreport if CV or PV is low, and not to underreport if it is high. Higher CV uncertainty makes this more difficult. Members with low PV can receive high CV signals, making the role of designated bidder more attractive. Also, a member with high PV and low CV signal perceives the role of the designated bidder as less attractive if the CV variance is higher.

If (4) holds, there exists a respective recommended bid function, $\beta(\cdot)$ which ring members follow, and which maximizes collusive gains. For an all-inclusive cartel, the existence is clear, a sufficiently high bid of the designated bidder deters other members from bidding higher. Proposition 2.3 extends this to non-inclusive cartels. The supremum of the set of best responses of the designated bidder to outsider strategies is a solution. The other bidders have lower valuation than the designated bidder. Bidding higher than the designated bidder's optimal bid results in a positive payoff if and only if the designated bidder could increase her own payoff by bidding higher. This contradicts best-response bidding.

Inequality (5) shows an explicit example for Proposition 2.3 with $n = 2$, $x_L = 0$, $x_H = 1$, $y_L = -z$ and $x_L = z$ with $z \geq 0$. This set of cases covers the normalization of the entire parameter space, and by z we can model the effect of CV information asymmetry by keeping the *ex ante* expected value of the CV term constant. As before, $\pi(\cdot)$ captures expected payoff of the designated bidder without side-payments, with a given information set. The first argument refers to the PV of the designated bidder, the second is the number of high CV signals of the ring. In the example, we write the probabilities explicitly using exogenous parameter δ, which expresses the quality of CV signals. Higher values mean better signals.

$$\left(\delta^2 + (1-\delta)^2\right) \pi\left(1, -z, -z, \alpha\left(\cdot\right)\right) + \left(1 - \delta^2 - (1-\delta)^2\right) \pi\left(1, z, -z, \alpha\left(\cdot\right)\right) \geq$$
$$\left(\delta^2 + (1-\delta)^2\right) \pi\left(0, z, z, \alpha\left(\cdot\right)\right) + \left(1 - \delta^2 - (1-\delta)^2\right) \pi\left(0, z, -z, \alpha\left(\cdot\right)\right) \tag{5}$$

The interpretation of our result stems from the relative importance of the PV as in the case of (4). Collusion is feasible, if payoff generated by high PV's is higher than the payoff for low PV with higher CV signals. If CV uncertainty decreases, in other words, z is lower, the LHS becomes relatively higher, making collusive agreements incentive compatible. Note that the value of z does not change the *ex ante* expected valuation of bidders.

In a first-price auction the designated bidder faces a threat that other ring members might outbid her. This threat results in a suboptimal collusive outcome. Let us see an example. Suppose there are two members of an all-inclusive ring with valuations equal to $x_1 = \frac{3}{2}$ and $x_2 = \frac{1}{2}$ in a pure PV auction with $z = 0$, so without CV uncertainty. Bidders truthfully reveal their types and make bidder 1 designated bidder. In a second-price auction after side-payments are paid, it is an equilibrium that bidder 1 submits 1, or any value higher than $\frac{1}{2}$ and bidder 2 submits 0. It is clear that they will comply with the agreement and they pay 0, maximizing the collusive gain. In a first-price auction, the ring is unable to achieve the first-best outcome. Player 2 only follows the recommended bid if bidder 1 submits more than $\frac{1}{2}$, similarly to the previous case, but in a first-price auction this results in a selling price $\frac{1}{2} > 0$.

This is reflected in the incentive compatibility constraints. In the last stage of the game, the designated bidder's expected payoff in the auction depends on the PV of the second highest PV in the ring. We denote this by function $\pi^*\left(x_1, \max x_{-1}, |y_H|, \alpha\left(\cdot\right)\right)$, which is analogously defined as $\pi\left(\cdot\right)$. The second argument denotes the highest opposing PV within the ring.

Proposition 2.4. *Suppose there is a given bidding ring with n members and $k \geq 0$ outsiders in a sealed-bid first-price auction. There exists a PNBE in which a bidding ring is formed, types are truthfully revealed (Condition* (1)*) and members comply with recommended bids (Condition* (2)*), if and only if constraint* (6) *is satisfied. In equilibrium, not all ring competition is suppressed.*

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_L\right)\frac{1}{i}\pi^*\left(x_1 = x_H, \max x_{-1}, |y_H| = j, \alpha\left(\cdot\right)\right) \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_H\right)\frac{1}{i+1}\pi^*\left(x_1 = x_L, \max x_{-1}, |y_H| = j, \alpha\left(\cdot\right)\right) \quad (6)$$

Proof. Constraint (6) is a necessary and sufficient condition for the existence of an incentive compatible mechanism. The derivation is identical to that of Proposition 2.3, but the expected payoff function is different, payoff depends on PV of other ring members. Function $\pi^*\left(\cdot\right)$ takes this into consideration, and expresses that expected payoff depends on the highest other PV.

□

There is no BCM. An incentive compatible PBNE means that ring members truthfully reveal their types (Condition 1) and they follow recommended bids (Condition 2). With a positive weight on the highest bid in the selling price function, the aggregate payoff of the ring is not maximal, Condition 3 is violated. The designated bidder shall increase her bid in order to avoid that other ring members violate the agreement by bidding higher. In contrast with second-price auctions, this increases the expected price conditional on winning, since the price is a strictly increasing function of the highest bid.

Suppose that all ring competition can be eliminated and the designated bidder bids her best response to outside bid functions (for an inclusive ring, that is 0). In that case, a non-designated bidder with identical PV can be better off by bidding marginally higher, obtaining a positive expected payoff. As such, the designated bidder's bid is higher than optimal. Positive *ex ante* payoff of the designated bidder is guaranteed by the positive probability of having no other ring member with identical PV. This argument also highlights why the expected revenue $\pi^*\left(\cdot\right)$ depends on the type of the highest opposing ring member.

This conclusion corresponds to Marshall and Marx (2007), who concluded that an equilibrium BCM in first-price auctions with individual PVs is not able to suppress all ring competition. Our model extends their results to all-inclusive rings and adds the insights regarding the feasibility of collusion if CV uncertainty is present.

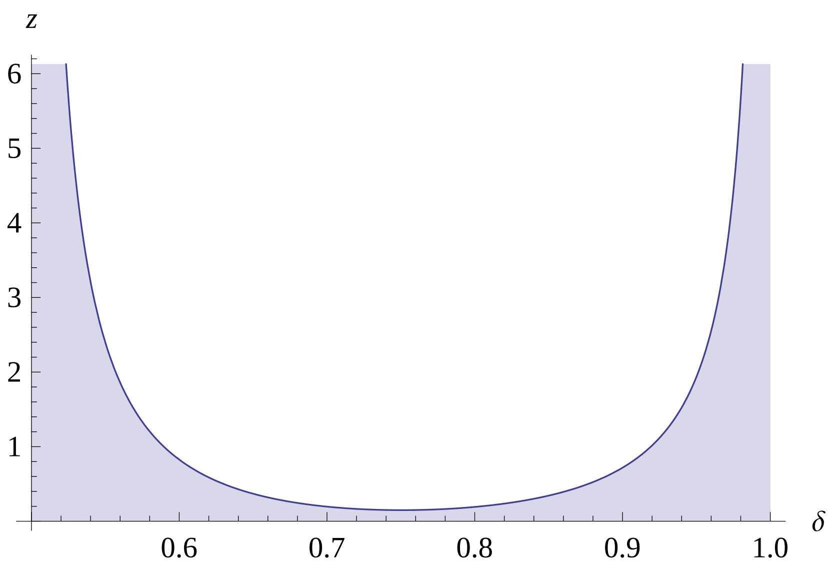

Figure 1: Existence of incentive compatible BCM, two ring members and one outsider, second-price sealed-bid auction.

3 Example: Second-price auctions

In order to motivate the intuition of Proposition 2.3, we illustrate our results for the case of second-price auctions. The incentive compatibility constraint (4) is quite general and the expected payoff function $\pi(\cdot)$ depends on multiple factors. In order to examine the existence of an incentive compatible BCM, an outsider bid function has to be specified. Subsection 3.1 concerns the non-cooperative equilibrium in a second-price sealed-bid auction. The existence of an incentive compatible BCM for the example is derived in Subsection 3.2.

3.1 Non-cooperative equilibrium

As before, we look for a pure-strategy BNE. A numerical example is provided in the Web Appendix for two ring members and one outsider bidder.[26]

There exists a unique symmetric BNE in a bigger class of auctions. Milgrom and Weber (1982) show the existence of a symmetric equilibrium in second-price sealed-bid auctions. Equilibrium bids satisfy that bidders are

[26] `http://gyulaseres.weebly.com/research.html`

indifferent between winning and not-winning where the highest opposing bid is identical. This solution comprises the pure PV equilibrium as a special case at which bidders submit their values. Levin and Harstad (1986) also demonstrate that this is the unique symmetric equilibrium.

In the example below, we consider again the case $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$, where $z \geq 0$. Let us determine the expected revenue of the seller in case of three bidders. Given the unique symmetric equilibrium strategy, this value can be determined by the probability distribution of the second highest bid (b_2^*). Now we focus on the case, where $b^*(1,-z) < b^*(0,z)$. The other one can be calculated accordingly. The probability of the second highest bid being equal to the highest possible value can be calculated in the following way. If this is the case, the two highest bids are both equal to $b^*(1,z)$.

The *ex ante* probability of high and low CV is $\frac{1}{2}$. Let us consider $y = z$, and calculate probabilities conditional on that. One can distinguish cases in which the lowest bid takes four different values. It is associated with the highest signal $(1,z)$ with probability $\frac{1}{8}\delta^3$. For signal $(1,-z)$, the probability of this being the lowest bid equals $\frac{3}{8}\delta^2(1-\delta)$, where numerator 3 refers to the three possible bidders having lower bids. Similarly, signals $(0,z)$ and $(0,-z)$ give probabilities $\frac{3}{8}\delta^3$ and $\frac{3}{8}\delta^2(1-\delta)$.

In case $y = -z$, conditional probabilities of high ($y_i = z$) and low ($y_i = -z$) signals are reversed, such that the lowest bid is maximal if all signals are $(1,z)$, which occurs with probability $\frac{1}{8}(1-\delta)^3$. Similarly, the lowest bid with signals $(1,-z)$, $(0,z)$ and $(0,-z)$ give probabilities $\frac{3}{8}\delta(1-\delta)^2$, $\frac{3}{8}(1-\delta)^3$ and $\frac{3}{8}\delta(1-\delta)^2$, respectively. Adding up probability values of the lowest value yields the result in equation (7).

$$
\begin{aligned}
Pr\left(b_2^* = b^*(1,z)\right) &= \frac{1}{16}\left(\delta^3 + 3\delta^2(1-\delta) + 3\delta^3 + 3\delta^2(1-\delta) + (1-\delta)^3\right. \\
&\quad \left. +\ 3\delta(1-\delta)^2 + 3(1-\delta)^3 + 3\delta(1-\delta)^2\right) \\
&= \frac{1}{8}\left(2 - 3\delta + 3\delta^2\right) \qquad (7)
\end{aligned}
$$

The probability of the lowest value being the second highest bid is identical. It takes the other two values with equal probabilities, $\frac{1}{8}(2 + 3\delta - 3\delta^2)$. This symmetry is implied by the fact that there are three bidders.

That is, expected revenue with 2 ring members and 1 outsider is expressed as:

$$\mathbb{E}R(z) = \frac{1}{8}\left[\left(2-3\delta+3\delta^2\right)\left(b^*(1,z)+b^*(0,-z)\right) + \left(2+3\delta-3\delta^2\right)\left(b^*(1,-z)+b^*(0,z)\right)\right] \tag{8}$$

3.2 Existence of a BCM

We show that the result of Milgrom and Weber (1982) about increasing revenue with respect to CV information asymmetry depends on the non-cooperative behavior of bidders, and does not hold if players are allowed to form a bidding ring. Collusion does not occur on the whole range of parameters. Since the cooperative and non-cooperative outcome differs, the expected revenue function is non-increasing and discontinuous. Throughout this setting we assume the participation of 2 ring members and 1 outsider to illustrate this point.

A crucial result for this conclusion is the set of parameters on which there is an incentive compatible BCM. Values of the side-payment function $\pi(\cdot)$ and the designated bid function $\beta(\cdot)$ depend on the order of non-cooperative equilibrium bids. While the highest value always occurs at information set $(1,z)$, and the lowest at $(0,-z)$, the order of the other two depends on the sign of q defined in equation (9), which is simply the difference of the two equilibrium bids in question.

$$q \equiv \frac{6-14\delta+12\delta^2-\delta^3}{3-7\delta+7\delta^2} + \frac{-3+7\delta-3\delta^2+2\delta^3}{3-7\delta+7\delta^2}z - \left(\frac{2\delta^2-\delta^3}{1-\delta+\delta^2} + \frac{1-\delta-3\delta^2+2\delta^3}{1-\delta+\delta^2}z\right) \tag{9}$$

Now we need to determine the designated bid, which maximizes the expected payoff of the player, conditional on truthfully revealed type. Here we can simplify further, since the x_2 is irrelevant here: ring member 2 submits a cover bid, so she does not affect the selling price. By parameter δ, the ring can calculate the conditional probability distribution of y and (x_3, y_3), and choose an optimal bid.

As a technical assumption, we add that ties are broken in favor of the designated bidder. Bids must be sufficiently high such that the non-designated player finds it optimal not to overbid. The optimal bids providing maximal payoff for the ring have interval-valued solutions. We choose the supremum of these intervals. For these values, it is always satisfied that the non-designated player does not find it profitable to overbid.[27] We can say that the designated ring member submits a value very close to the non-cooperative equilibrium bids.

The solution follows equation (10) for the unique symmetric equilibrium in second-price sealed-bid auction if $b^*(0,z) \leq b^*(1,-z)$ and (11). If the opposite is true, $b^*(0,z) \geq b^*(1,-z)$ occurs.

$$\beta(\cdot) = \begin{cases} b^*(1,z)+1, & \text{if } x_1 = 1,\ y_1 = y_2 = z; \\ b^*(1,z), & \text{if } x_1 = 1,\ y_1 \neq y_2; \\ b^*(1,-z), & \text{if } x_1 = 1,\ y_1 = y_2 = -z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 \neq y_2; \\ b^*(0,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = -z; \end{cases} \tag{10}$$

$$\beta(\cdot) = \begin{cases} b^*(1,z)+1, & \text{if } x_1 = 1,\ y_1 = y_2 = z; \\ b^*(1,z), & \text{if } x_1 = 1,\ y_1 \neq y_2; \\ b^*(0,z), & \text{if } x_1 = 1,\ y_1 = y_2 = -z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = z; \\ b^*(1,-z), & \text{if } x_1 = 0,\ y_1 \neq y_2; \\ b^*(0,-z), & \text{if } x_1 = 0,\ y_1 = y_2 = -z; \end{cases} \tag{11}$$

Two possible versions of inequality (5) can be found in the Web Appendix. The parameter set on which there is an incentive compatible BCM is depicted in Figure 1. There exists a BCM for points of the shaded area in the space of (z,δ). For all $\frac{1}{2} < \delta < 1$, there exists an incentive compatible BCM, if z is sufficiently low. Equation (9) defines the switch between the two parametric forms.

As an illustration, let us consider a few examples.[28] We apply the notation $\bar{z}(\delta)$ for the supremum of the set on z given δ on which collusion is feasible. If $\delta = 0.6$, there is an incentive compatible BCM if and only if

[27] In case of information set $(x_1 = 1, y_1 = y_2 = z)$, the set has no supremum, any bid higher than the highest possible outside bid is optimal. For example, $1+z$ is optimal, since it is the highest possible valuation of any bidder.

[28] Values are calculated and figures are created by Wolfram Mathematica. See Web Appendix.

$$z \geq \bar{z}\,(0.6) \approx 0.826109$$

For the example above, there is a critical value of $\bar{z}\,(\delta)$ for every $\delta \in \left(\frac{1}{2}, 1\right)$ such that there is an incentive compatible BCM for a given signal quality δ if and only if $z \leq z\,(\delta)$.

The example above supports the claim that the existence of an incentive compatible mechanism depends on the proximity of the pure private model, which has a neighborhood satisfying this criterion on the whole range of parameter δ. Less available information (higher z) about the commodity makes collusion infeasible.

4 Perturbed games and robustness

Proposition 2.3 appears to be robust against perturbations in information asymmetry. Perturbed games in which private information is introduced to a CV model focus on the effect of private information on the bidding behavior of the informed bidder in affiliated (Klemperer, 1998) and non-affiliated (Larson, 2009) settings. Perturbation can be used to examine the robustness of our conclusions with respect to the pure models.

The significance of these results is highlighted by the expected revenue function, for which we provide an example in Section 5. If the existence of BCM is robust with respect to perturbations for pure private and CV models, there exists an interior cut-off point. That is, if we consider a range of settings with respect to CV uncertainty by keeping everything else constant, including *ex ante* expected CV, there is an interior point at which the existence of BCM changes. If CV uncertainty decreases, it induces a positive downward jump in expected revenue. That is, expected revenue is not an increasing function of the availability of public information, in contrast with the non-cooperative model of Milgrom and Weber (1982).

The neighborhood of pure PV models supports collusion. On the contrary, the neighborhood of pure CV models does not. Lemma 4.1 concerns robustness of the pure PV model. In order to capture CV uncertainty, we apply the normalization $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$. Outsider strategy is denoted as $\alpha\,(x_{out}, y_{out}, z)$, where x_{out} and y_{out} are the signals observed by outsiders.

Lemma 4.1. *Suppose a bidding ring is formed in a second-price sealed-bid auction in which the selling price is independent of the highest bid. Assume that* $\alpha\left(x_{out}, y_{out}, z\right)$ *is continuous with respect to* y_{out} *at a neighborhood of* $(x=0, z=0)$. *Then, there exists a right-side neighborhood of* 0 *on the range of* z, *on which there is an incentive compatible BCM.*

Proof. First, we point out that continuity of outsider strategy $\alpha\left(\cdot\right)$ implies that $\pi\left(\cdot\right)$ is also continuous at a neighborhood of $(x_1 = 0, |y_i = 0| = i)$ for any $i \in \{0, \ldots, n\}$, since the price function is a linear combination of bids. Let us consider (4) and substitute $z = 0$. Then the inequality simplifies to

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i\right)\frac{1}{i+1}\pi\left(x_1 = 1, \cdot\right) \geq$$
$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i\right)\frac{1}{n-i}\pi\left(x_1 = 0, \cdot\right) \tag{12}$$

However, this is always satisfied. Consider the optimal strategy for information set $x_i = 0$. The same strategy yields higher payoff for information set 1, and strictly higher, if there is a positive probability of winning with 0. The latter condition is relevant for the existence of the above-defined neighborhood. If the designated player also bids according to the outsider strategy, that provides a positive probability of winning with a non-negative payoff, so these results apply to the optimal choice as well. Thus, for any sealed-bid auction, constraint (12) is satisfied as a strict inequality. With continuity of $\pi\left(\cdot\right)$, we can conclude.

□

Lemma 4.1 shows that, in accordance with earlier findings, the pure PV model always supports collusion. Moreover, the result is robust to small CV perturbations. In other words, for low levels of common uncertainty, an incentive compatible bid coordination mechanism is always sustained.

This result also holds for the opposite direction, as it is formalized in Lemma 4.2. Here we examine the environment of the point $x_L = 1$, $x_H = 1 + \varepsilon$, $y_L = -z$ and $y_H = z$ in order to examine the neighborhood of the pure CV model.

Lemma 4.2. *Suppose a bidding ring is formed with* $n \geq 2$ *members and there are* $k \geq 0$ *outsiders in a second-price auction. Assume that* $\alpha\left(x_{out}, y_{out}, z\right)$ *is* *continuous in the neighborhood of $(x_i = 1, |x_H| = i)$ for any $i \in \{0, \dots, n\}$. Then, for every $\delta \in \left(\frac{1}{2}, 1\right)$, there exists a right-side neighborhood of* 1 *on the range of x_H, on which there is no BCM.*

Proof. The proof is analogous to Lemma 4.1. □

Lemma 4.1 and 4.2 has an important implication, expressed in Corollary 4.3 in terms of x_L, x_H, y_L and y_H. For any given $x_L \neq x_H$, CV uncertainty is defined as $y_H - y_L$.

Corollary 4.3. *Suppose the assumptions of Lemma 4.1 and 4.2 hold. Then, there exists a BCM in a pure PV model ($y_L = y_H$) and there exists no BCM in the neighborhood of the pure CV model ($x_L + \varepsilon = x_H$). Moreover, there exists an interior cutoff point on the set of CV uncertainty with respect to the existence of BCM.*

Proof. The set $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$ with $z \geq 0$ and $x_H > 0$ is a normalization of the set above. We address the existence of BCM on the domain of z. Following Lemma 4.1, there exists a BCM if z is sufficiently close to 0, thus, the model is close to IPV. Similarly, following Lemma 4.2, there exists no BCM if z is sufficiently high. □

This result is illustrated in Section 5. We note that analogous results can be derived for first-price auctions. To illustrate this, Constraint (6) simplifies to (13).

$$\sum_{i=0}^{n-1} \sum_{j=1}^{n} Pr\left(|x_H| = i\right) \frac{1}{i+1} \pi^*\left(x_1 = 1, \max x_{-1}, \cdot\right) \geq$$
$$\sum_{i=1}^{n} \sum_{j=0}^{n-1} Pr\left(|x_H| = i\right) \frac{1}{n-i} \pi^*\left(x_1 = 0, 0, \cdot\right) \tag{13}$$

5 Expected revenue and CV variance

The negative effect of CV uncertainty in non-cooperative auctions is a robust result. Less uncertainty results in higher expected revenue. We illustrate that this is not the case for a collusive setting. Lower common uncertainty makes collusion incentive compatible. Hence, it decreases revenue on a part of the domain.

Milgrom and Weber (1982) state that publicly revealed information has a non-negative effect on expected revenue. Goeree and Offerman (2003) have the same conclusion with non-affiliated values. It is also supported by experimental (Goeree and Offerman, 2002; Kagel et al., 1995) findings. Also, Silva et al. (2008) found the same conclusion in an empirical model directly testing the effect of greater public information in procurement auctions.

Expression (14) determines the first-order derivative of the expected revenue with respect to z in second-price auctions with 3 bidders, for all $\delta \in \left(\frac{1}{2}, 1\right)$, for the normalized case $x_L = 0$, $x_H = 1$, $y_L = -z$ and $y_H = z$.[29]

$$\frac{\partial \mathbb{E}R(z)}{\partial z} = -\frac{4(\delta-1)^2 \delta^2 (2\delta-1)(-3+12\delta-20\delta^2+10\delta^3+10\delta^4-18\delta^5+6\delta^6)}{(1-\delta+\delta^2)(1-3\delta+3\delta^2)(3-5\delta+5\delta^2)(3-7\delta+7\delta^2)} \tag{14}$$

The derivative $\frac{\partial \mathbb{E}R(z)}{\partial z}$ is always negative on the range of δ. Thus, the expected revenue is decreasing in z. In other words, lower variance has a positive effect on the expected revenue and $z = 0$ provides the highest possible revenue for any given δ. This is consistent with earlier findings cited above, lower CV variance results in higher expected revenue.

Figure 2 depicts the marginal effect for all possible δ. It can be noted that the effect of CV uncertainty diminishes for extreme values of δ, which determines the probability of a CV signal being correct. First, if δ is close to $\frac{1}{2}$, the CV signal is nearly pure noise. If it is close to 1, the signal is almost perfect, common uncertainty has again no role in the limit.

This result is ambiguous if bidders are able to collude. At $z = \bar{z}(\delta)$, BCM decreases the expected revenue, and the expected revenue function is discontinuous at that point. If there is no incentive compatible BCM, potential ring members correctly anticipate that type signals are not credible, so they do not form a ring.[30] Thus, players bid according to the unique symmetric BNE, and expected revenue follows equation (8). If there is an incentive compatible BCM, they engage in a collusive agreement. The expected revenue of this case depends also on the sign of q. Derivations can be found in the Web Appendix.

Let us consider an example, and set $\delta = 0.6$. The expected revenue function $\mathbb{E}R(z)$ is depicted in Figure 3 for the case when the seller is able

[29] We get the expected revenue from the non-cooperative equilibrium strategies and the probability distribution of the types of bidders.

[30] This is a standard assumption in the literature (Marshall and Marx, 2009).

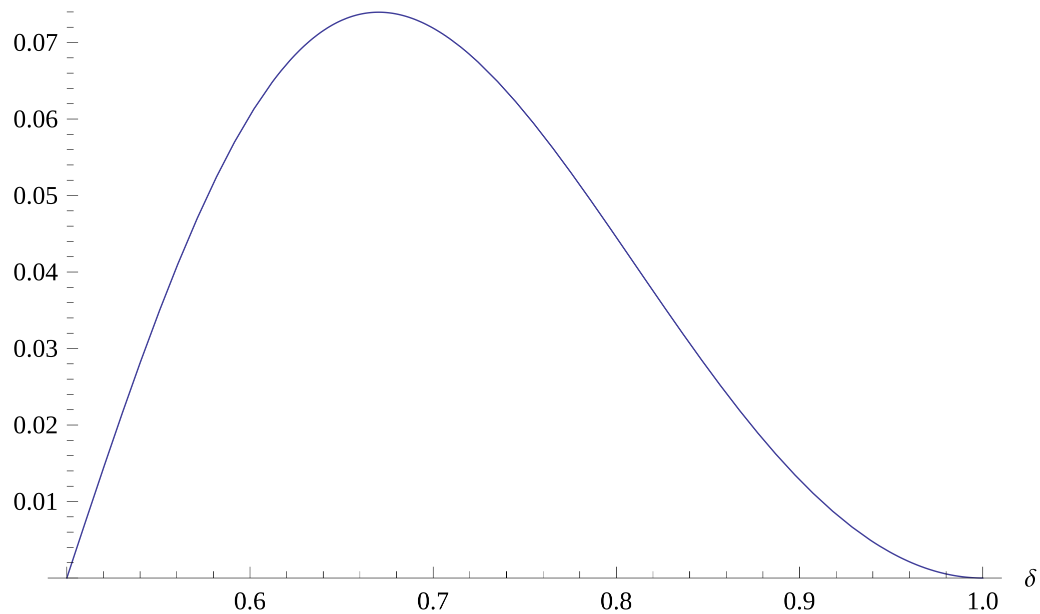

Figure 2: Marginal effect of CV uncertainty on expected revenue, with competing bidders

to reduce z to 1. In the non-cooperative outcome, revenue is a decreasing function of z. Collusion is feasible, if $z \geq \bar{z}(0.6) \approx 0.826109$. As such, ring members can engage in a collusive mechanism at $\bar{z}(0.6)$, which decreases the revenue, and results in a discontinuous function.

Discontinuity is a consequence of the lack of enforced collusive agreement for high z. Collusion does not always emerge if ring members are able to increase their payoff. They also need to provide sufficient incentives by side-payments to prevent the misreporting of types. Consequently, at the point by which sufficient information is revealed to form a bidding ring, they have a strictly positive gain, resulting in a discontinuity point of the expected revenue function. This is an interior point, followed by Proposition 4.3.

However, revenue is decreasing with respect to z to the right from the point of discontinuity as well. So, this is the only point on the domain at which $\mathbb{E}R(z)$ is non-increasing or discontinuous. The negative slope to the right of $\bar{z}(0.6)$ has a similar intuition as the non-cooperative outcome. If a bidding ring is formed, the number of bidders is reduced to two, which are informed asymmetrically. Lower values of z reduce the common uncertainty,

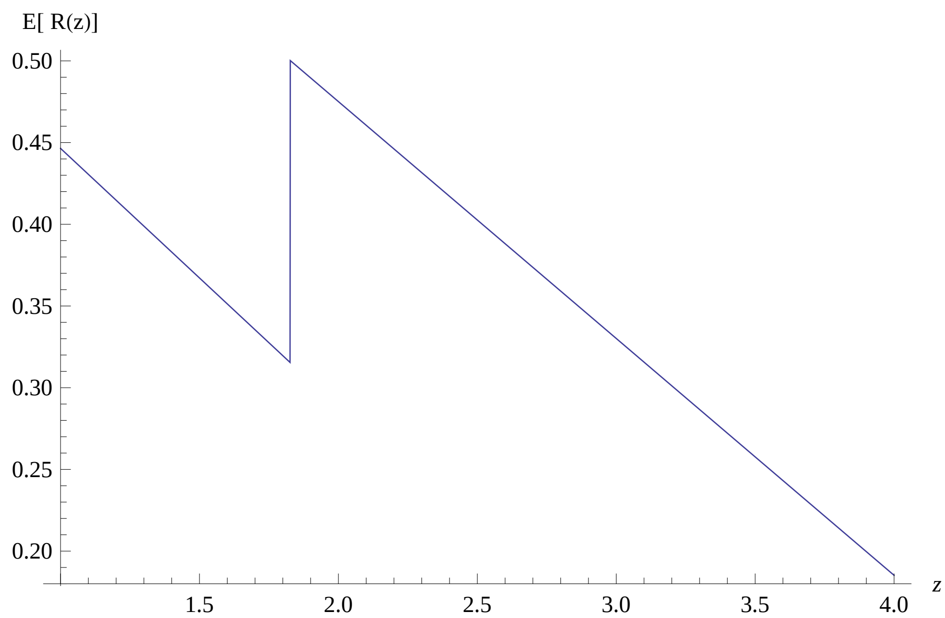

Figure 3: Expected revenue as a function of z, $\delta = 0.6$

and increase the lower bids, resulting in higher expected revenue.

Consequently, a revenue-maximizing seller might not find it optimal to reduce common uncertainty about the commodity. We can expect that it is not possible to completely eliminate common uncertainty, so that the seller is only able to choose from a constrained set. In Figure 3 this is illustrated, full disclosure is not optimal. Since expected revenue is not a monotonic function of CV uncertainty, an interior solution might be optimal. While lower variance increases the expected revenue conditional on non-cooperative behavior, it enhances cartel stability and might lead to lower revenue, as in our numerical example.

6 Conclusion

The role of common value uncertainty in cartel coordination is a crucial one. Private information about market demand can destroy collusive equilibria (Kandori and Matsushima, 1998). Variance of the stochastic demand component decreases the excess profit of a cartel (Porter, 1983). Theoretical literature emphasizes that common uncertainty weakens the effect of punishment mechanisms. Our paper, focusing on an auction setting, adds the notion that the latter effect can hold for cartels without repeated interaction or punishment.

Collusion in auctions is a prevalent phenomenon.[31] Related theory literature addresses the role of a strategic seller, emphasizing the significance of information disclosure. This paper contributes to the debate by adding that reducing uncertainty about the commodity in a collusive market can be damaging. In some cases, it can help cartel stability and reduce revenue.

This paper builds up an auction model with additively separable common and private value elements, and symmetric, risk-neutral bidders. Moreover, we assume that an exogenous subset of bidders can engage in a collusive agreement without the possibility of enforced bids. Milgrom and Weber (1982) prove that revenue is a non-decreasing function of the publicly available information if players bid competitively, known as the Linkage Principle. We conclude, that information revelation supports collusion and can result in a negative effect on revenue. This result is robust for sealed-bid auction mechanisms. Reducing common uncertainty by the seller helps to sustain collusive mechanisms, which reduces expected revenue. For sealed bid auctions in which price is not increasing in the highest bid, most notably in second-price auctions, this drop occurs for a partial reduction of uncertainty. That is, expected revenue is not a monotonic function of common value variance in collusive auctions. Consequently, a seller who is unable to completely eliminate common value uncertainty might find it optimal to partially reduce it. That is, the Linkage Principle fails if collusion can occur.

Our paper contributes to the collusion literature. Our model implies that the effect of reducing information asymmetry is ambiguous, similarly to the case of production markets. It helps reduce uncertainty, and results in more efficient outcomes. On the other hand, it might sustain coordination of colluding players if it signals compliance with a collusive agreement. What our paper adds is that the collusion inducing effect persists even if it does not help coordination. Lower uncertainty about valuations helps sustain cartels by increasing incentive compatibility of collusive mechanisms. As a result, it is able to reduce expected revenue in auctions. We can conclude that separating the effect of private and common value information asymmetry can help us better understand the mechanism behind this phenomenon.

[31] Marshall et al. (2014) cite a number of recent bid rigging cases by the US Department of Justice.

Appendices

A Existence of BCM

First, we derive the necessary and sufficient conditions for the existence of a BCM, which satisfies truthful revelation, and such that bidders comply with recommended bids. Then, we derive that this is sufficient for the existence of a mechanism which also maximizes the aggregate expected payoff of the ring.

A bidder has four possible information sets depending on the private and CV types. The side-payment $p\left(|x_H|, |y_H|\right)$ is a function of these reports. This is the total amount that members with low PV receive if there is at least one member with high reported CV. The arguments $|x_H|$ and $|y_H|$ refer to the number of high reported PVs and CVs among ring members. Thus, $p\left(|x_H|, |y_H|\right)$ can take $(n+1)^2$ values. We apply the notation $p\left(|x_H|, |y_H|\right) = p_{|x_H|,|y_H|}$. For example, $p_{1,2}$ refers to the total side-payment player with low PV receives if the number of high private and CV signals among members is 1 and 2, respectively. Thus, the side-payment paid each player with high PV amounts to $\frac{1}{|x_H|}p_{|x_H|,|y_H|}$ and each ring member with low PV receives $\frac{1}{n-|x_H|}p_{|x_H|,|y_H|}$.

Each of the four information sets defines three incentive compatibility constraints determining that the ring member is better off with a truthfully revealed type, than with a misreport. In each case, $\pi\left(x_1, |y_H| = j, \alpha\left(\cdot\right)\right)$ denotes the expected payoff of the designated bidder (without loss of generality, member 1) who participates in the auction, depending on the learned PV type and the number of high value signals of the ring, not taking side-payments into account. The constraints are applied to unilateral deviations. Thus, the misreporting member learns the type of the other ring members, and these expected profits are unconstrained.

The 12 incentive compatibility constraints (ICC) are indexed. As before, L refers to low and H refers to high values. For example, in ($ICC_{LL,LH}$), the first two characters show the information set of the bidder, low PV and low CV signal. The last two denotes the misreport in the same order, low private and high CV.

First we consider the four constraints at which only the CV signal is misreported. A misreport of this type does not affect the choice of the designated bidder as derived in Lemma 2.1, so that we can simplify the expected designated revenue term $\pi(\cdot)$. On the other hand, it can change the side-payments, since it depends on the number of high CV reports, as derived above. In $(ICC_{LL,LH})$, the bidder can only receive side-payment, never pays it, since the PV report is low on both sides. The side-payments depend on the number of CV reports, which is altered on the right hand side (RHS). The probability distribution remains the same, since it refers to the distribution of types of the other ring members. All four constraints are calculated analogously. It can be noted that the expected payoff function $\pi(\cdot)$ does not appear here, since CV misreport does not change the choice of the designated bidder. That is, it cancels out in all four cases. In constraints $(ICC_{HL,HH})$ and $(ICC_{HH,HL})$, negative signs are explained by that the player reports high PV.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_L\right)\frac{1}{n-i}p_{i,j} \quad \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_L\right)\frac{1}{n-i}p_{i,j+1} \qquad (ICC_{LL,LH})$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_H\right)\frac{1}{n-i}p_{i,j} \quad \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_H\right)\frac{1}{n-i}p_{i,j-1} \qquad (ICC_{LH,LL})$$

$$-\sum_{i=1}^{n}\sum_{j=0}^{n-1}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_L\right)\frac{1}{i}p_{i,j} \quad \geq$$
$$-\sum_{i=1}^{n}\sum_{j=0}^{n-1}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_L\right)\frac{1}{i}p_{i,j+1} \qquad (ICC_{HL,HH})$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{n}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_H\right)\frac{1}{i}p_{i,j} \quad \geq$$
$$-\sum_{i=1}^{n}\sum_{j=1}^{n}Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_H\right)\frac{1}{i}p_{i,j-1} \qquad (ICC_{HH,HL})$$

Let us take inequality ($ICC_{HL,HH}$) and multiply it by -1. This way both the LHS and RHS of constraints ($ICC_{LL,LH}$) and ($ICC_{HL,HH}$) are identical. The same holds for ($ICC_{LH,LL}$) and ($ICC_{HH,HL}$). That is, if all ICCs hold, all of them are binding, they hold with equality.

The remaining eight constraints address deviations involving misreported PV affecting winning probabilities. Firstly, we point out, that these inequalities can be grouped into four pairs, which are pairwise identical if the constraints above involving only CV misreport hold. This point is demonstrated for the cases of $ICC_{LL,HL}$ and $ICC_{LL,HH}$. The LHS of the constraints are identical, whereas the RHS is different by the report of the CV. The designated player's payoffs and respective probabilities are the same, since in both cases the PV reports and the information sets are identical. The side-payments are different, but if the first four constraints above hold, $ICC_{HH,HL}$ is binding. Accordingly, if all constraints are satisfied, $ICC_{LL,HL}$ and $ICC_{LL,HH}$ are identical. Similarly, this result holds for pairs ($ICC_{LH,HL}, ICC_{LH,HH}$), ($ICC_{HL,LL}, ICC_{HL,LH}$) and ($ICC_{HH,LH}, ICC_{HH,LL}$).

Consequently, in order to consider incentive compatibility, we only need to examine $ICC_{LL,HL}$, $ICC_{LH,HL}$, $ICC_{HL,LH}$ and $ICC_{HH,LH}$.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_L, y_1 = y_L\right) \frac{1}{n-i} p_{i,j} \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_L, y_1 = y_L\right) \frac{1}{i+1} \left(\pi\left(x_1 = x_L, |y_H| = j, \alpha(\cdot)\right) - p_{i+1,j}\right) \qquad (ICC_{LL,HL})$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_L, y_1 = y_H\right) \frac{1}{n-i} p_{i,j} \geq$$
$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_L, y_1 = y_H\right) \frac{1}{i+1} \left(\pi\left(x_1 = x_L, |y_H| = j, \alpha(\cdot)\right) - p_{i+1,j-1}\right) \qquad (ICC_{LH,HL})$$

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_H, y_1 = y_L\right) \frac{1}{i} \left(\pi\left(x_1 = x_H, |y_H| = j, \alpha(\cdot)\right) - p_{i,j}\right) \geq$$
$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i, |y_H| = j \,\middle|\, x_1 = x_H, y_1 = y_L\right) \frac{1}{n-i+1} p_{i-1,j+1} \qquad (ICC_{HL,LH})$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_H\right)\frac{1}{i}\left(\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)-p_{i,j}\right) \geq$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_H\right)\frac{1}{n-i+1}p_{i-1,j} \qquad (ICC_{HH,LH})$$

Let us rearrange the constraints according to side-payment components. It can be seen that they are all equal. Here we use that the probability coefficients follow a binomial distribution, which is implied by that PV types are independent and CV types are conditionally independent.[32] As such, it is symmetric. This value is denoted by P in equation (15), while probability values are expressed explicitly. Let θ be a short-hand notation for the probability that another CV signal equals a bidder's own CV signal, such that $\theta=\delta^2+(1-\delta)^2$.

$$\begin{aligned}
&\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} \Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_L\right)\left(\frac{1}{n-i}p_{i,j}+\frac{1}{i+1}p_{i+1,j}\right) &=\\
&\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_L,y_1=y_H\right)\left(\frac{1}{n-i}p_{i,j}+\frac{1}{i+1}p_{i+1,j-1}\right) &=\\
&\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_L\right)\left(\frac{1}{n-i+1}p_{i-1,j+1}+\frac{1}{i}p_{i,j}\right) &=\\
&\sum_{i=1}^{n}\sum_{j=1}^{n} Pr\left(|x_H|=i,|y_H|=j \,\middle|\, x_1=x_H,y_1=y_H\right)\left(\frac{1}{n-i+1}p_{i-1,j}+\frac{1}{i}p_{i,j}\right) &=\\
&\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\binom{n}{i}\left(\frac{1}{2}\right)^n\binom{n}{j}(1-\theta)^j\theta^{n-j}\left(\frac{1}{n-i}p_{i,j}+\frac{1}{i+1}p_{i+1,j}\right) &=P
\end{aligned}$$

Equation (15) implies that the constraints $ICC_{LL,HL}$, $ICC_{LH,HL}$, $ICC_{HL,LH}$ and $ICC_{HH,LH}$ can be summarized according to (15) and (16), with explicit representation of probabilities.

[32]For instance, note, that $\Pr(y_j=y_L|y_i=y_L)\cdot\Pr(y_k=y_L|y_i=y_L)=\Pr(y_k=y_j=y_L|y_i=y_L)$.

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}(1-\theta)^{j}\theta^{n-j}\frac{1}{i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)\geq$$

$$P\geq$$

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}(1-\theta)^{j}\theta^{n-j}\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right) \quad (15)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}\theta^{j}(1-\theta)^{n-j}\frac{1}{i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)\geq$$

$$P\geq$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}\theta^{j}(1-\theta)^{n-j}\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right) \quad (16)$$

Constraints (15) and (16) have a solution if and only if inequality (17) holds. They define two upper and two lower bounds for the possible values of P. It can be noted that the LHS of (15) is smaller than the LHS of (16), because the CV signals are higher in the latter expression. A similar result applies to the RHS of both, CV signals are lower on the RHS of (15). As such, bounds are the tightest when the LHS of (15) and the RHS of (16) are equal. In order to have a solution for P, the latter must not be greater than the former. This way, we have defined a necessary and sufficient condition for the existence of an incentive compatible collusive equilibrium.

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=0}^{n-1}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}(1-\theta)^{j}\theta^{n-j}\frac{1}{i}\pi\left(x_1=x_H,|y_H|=j,\alpha\left(\cdot\right)\right)&\geq\\ \sum_{i=0}^{n-1}\sum_{j=1}^{n}\binom{n}{i}\left(\frac{1}{2}\right)^{n}\binom{n}{j}\theta^{j}(1-\theta)^{n-j}\frac{1}{i+1}\pi\left(x_1=x_L,|y_H|=j,\alpha\left(\cdot\right)\right)& \quad (17)\end{aligned}$$

Equivalently, the same result can be written as in inequality (18).[33]

[33] Identical to (4).

$$\sum_{i=1}^{n}\sum_{j=0}^{n-1} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_L\right) \frac{1}{i}\pi\left(x_1 = x_H, |y_H| = j, \alpha\left(\cdot\right)\right) \geq$$

$$\sum_{i=0}^{n-1}\sum_{j=1}^{n} Pr\left(|x_H| = i, |y_H| = j \;\middle|\; y_1 = y_H\right) \frac{1}{i+1}\pi\left(x_1 = x_L, |y_H| = j, \alpha\left(\cdot\right)\right) \quad (18)$$

What remains is to prove that the latter condition guarantees that no other bidder finds it profitable to outbid the designated bidder in a second-price auction. Again, without loss of generality, the designated bidder is denoted by 1. In order to bid optimally, we need that

$$\beta_1 = \arg\max \left[\sum \left[v_1 - \max\left\{\alpha\left(x_N, y_N\right)\right\}\right] Pr\left(\beta_1 > \max\left\{\alpha\left(x_N, y_N\right)\right\}\right)\right]$$

where $Pr\left(\beta_1 > \max\left\{\alpha\left(x_N, y_N\right)\right\}\right)$ is the probability that the designated bid is higher than the maximal outsider bid, given the information set $\left(x_N, y_N\right)$ of the ring. The types are truthfully reported, so that the distribution of valuations of the designated bidder and other ring members only differ in the PV. There are two cases. The PV of the other member with the highest PV is either lower or identical to the designated bidder's value. It is clear that it is enough to consider the latter case.

Consider the supremum of the set of optimal designated bids.[34] If deviation is profitable for a non-designated member, it would also be profitable to the designated bidder, contradicting that she submitted an optimal bid. In that case, the expected payoff of the non-designated bidder is less than or equal to the excess expected payoff the designated bidder could receive with the same bid, since the list of bids affecting price is strictly greater for the designated bidder, and the designated player's valuation is weakly greater.

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[34]In some cases, it has no supremum, we can set the designated bid to any value greater than $a_2\left(x_H + y_H\right)$, making deviation never profitable.

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